Development of Closed Formula of Wave Load Based Upon Long-Term Prediction[*1]

— Heave Acceleration and Pitch Angle —

Kyohei SHINOMOTO[*], Sadaoki MATSUI[**], Kei SUGIMOTO[*], Shinsaku ASHIDA[***]

1. INTRODUCTION

Ship designers need to be able to reasonably estimate hull motion in waves from various aspects such as passenger safety, ride comfort and wave loads for ship designs. Nowadays, hull motion in waves is accurately estimated by seakeeping analysis tools such as the 3-D panel method and the estimation is practically used for all hull design and standard development [1]. At the same time, however, there also is high demand for methods which allow for more simple estimations of hull motion to be made that do not rely on numerical analysis. For example, in wave load estimations for structural strength evaluations, it can be quite difficult to perform a wave load analysis for each ship due to the amount of time it would add to the hull structure design process. For this reason, a simple estimation method based upon a simple wave load formula using main ship parameters is typically adopted for classification society rules [2] [3].

Many prior studies of this matter have estimated the maximum loads of ships through statistical prediction of hull response in irregular waves using the energy spectrum method that applies the theory of linear superposition [4] [5]. The linear term of the maximum load specified in some classification society rules [2] [3] is also specified to be equivalent to the long-term predicted value of exceedance probability of 10^{-8}. Based upon this, it can be said that long-term prediction is an established method for estimating the linear term of the maximum load.

Aim of our larger study the development of a general-purpose and high precision closed formula for maximum loads with an exceedance probability of 10^{-8} by formulating the long-term prediction for ships of any size or type. A past study by Kawabe et al. [6] had a similar purpose but proposed a method for predicting maximum loads based upon structural analysis using calculation results obtained through a strip method and the long-term prediction of stress. Moreover, since the Kawabe et al. and Shigemi et al. [7] study made no attempt to formulate the standard deviation of the hull response in irregular waves. In addition, the study only focused on bulk carriers and oil tankers, it is difficult to use its results to guarantee the accuracy for other types and sizes of ships. This study, on the other hand, takes into account the standard deviation of the hull response in irregular waves and the directional distribution of irregular waves. Main ship parameters such as ship length L, breadth B, draft d, block coefficient C_b,, and water line area coefficient C_w were used for formulating the long-term prediction. Dominant factors and their trends were examined step by step and the formula that does not limit target types and sizes of ships was developed. The accuracy of this formula was then confirmed through numerical calculations using a model to represent any type and size of ship. We believe that in this manner we were able to develop a general-purpose and high precision closed formula for practical use.

This paper focused on heave acceleration and pitch angle. Since internal loads, etc. in the structural rules require an inclined component due to rotational motion, an angle was focused on for pitch motion.

Response amplitude operator (hereinafter, referred to as "RAO") obtained from a numerical calculation were used for RAO contained in the closed formula. The closed formula of RAO proposed by Jensen et al. [8] and Matsui et al. [9] can be used as RAO contained in the closed formula.

A flow diagram for obtaining a long-term predicted value with an exceedance probability of 10^{-8} is shown in Fig. 1. The long-term prediction can be divided into components, and these components are expressed as formulae explained in Sec. 4 onwards. The long-term prediction was formulated by developing these components using theoretical approaches as much as possible. Although we found it difficult to mathematically formulate some components, we were able to formulate them by

---

[*1] This article was announced at the 39th International Conference on Offshore Mechanics and Arctic Engineering (OMAE 2020) held in June 2020.

[*] Hull Rules Development Department, NIPPON KAIJI KYOKAI (ClassNK)
[**] National Maritime Research Institute
[***] Kawasaki Heavy Industries, Ltd. temporary affiliated with NIPPON KAIJI KYOKAI (ClassNK)

taking advantage of the diversity and total number of vessels used for the numerical calculations.

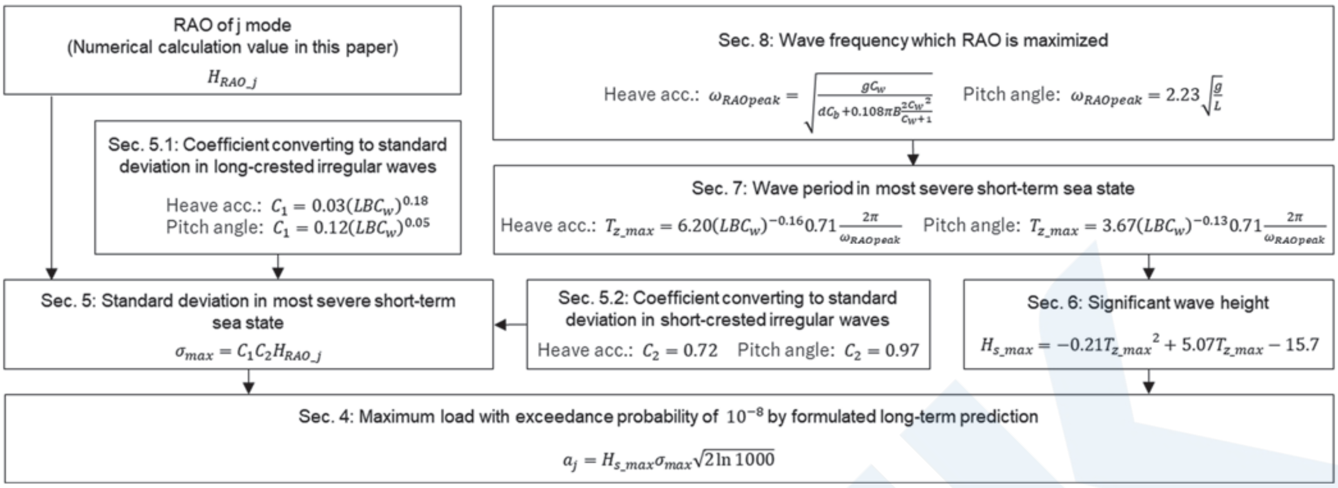
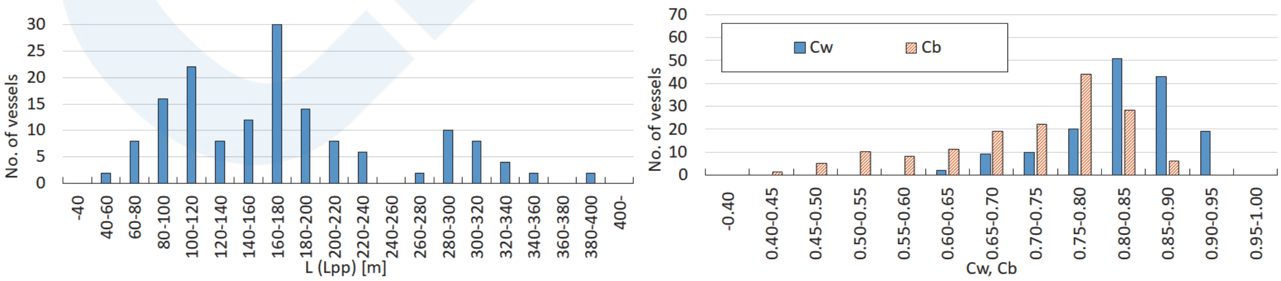


Figure 1 Flow diagram for formulated long-term prediction

2. NUMERICAL CALCULATIONS OF LINEAR ANALYSIS

In order to confirm the accuracy of our proposed closed formula, a numerical calculation with linear seakeeping analysis code developed by ClassNK was performed. The linear code is a 3-dimensional code based upon the Green function method which takes the forward speed effect approximation method proposed by Papanikolau et al. into account [10) 11)]. Sugimoto et al. validated the accuracy of this code by towing tank tests [1)]. The responses, i.e. motions and hull girder bending moment, calculated by this code and the values measured under the low-wave-height condition were in good agreement. However, it should be understood that there are some limitations to this code, and that these limitations may cause a slight discrepancy between the experimental and calculated values. One such example is the error between the experimental and calculated values that is expected to result from the fact that the panel size of the calculation model is relatively rough with respect to wavelength.

One hundred fifty-four models (77 existing ships × 2 loading conditions per ship) were used for the numerical calculation. Since the formula is intended to be applied to only general merchant ships, the target ships were limited to monohulls that had symmetrical shapes below their waterlines. As shown in Fig. 2, various types of vessels (such as bulk carriers, container carriers, wood chip carriers, general cargo carriers, liquefied gas carriers (LNG, LPG), ore carriers, oil tankers, vehicle carriers, and refrigerated carriers) were used so as to cover a wide range of values for L, C_b and C_w. In addition, forward speed was set at five knots in consideration of the decrease in ship speed with respect to high wave height, in reference to the Common Structural Rules for Bulk Carriers and Oil Tankers of the International Association of Classification Societies (hereinafter referred to as “IACS”) [12)].



(A) No. of Vessels by Length (B) No. of Vessels by C_w and C_b

Figure 2 Histogram of vessels used for confirmation

3. SEA STATE CONDITIONS

As shown in Fig. 3, the scatter diagram in IACS Recommendation No. 34 [13)] was used for sea state conditions.

Hs/Tz	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5	17.5	18.5	SUM
0.5	0	0	1.3	133.7	865.6	1186	634.2	186.3	36.9	5.6	0.7	0.1	0	0	0	0	0	0	3050.4
1.5	0	0	0	29.3	986	4976	7738	5569.7	2375.7	703.5	160.7	30.5	5.1	0.8	0.1	0	0	0	22575.4
2.5	0	0	0	2.2	197.5	2158.8	6230	7449.5	4860.4	2066	644.5	160.2	33.7	6.3	1.1	0.2	0	0	23810.4
3.5	0	0	0	0.2	34.9	695.5	3226.5	5675	5099.1	2838	1114.1	337.7	84.3	18.2	3.5	0.6	0.1	0	19127.7
4.5	0	0	0	0	6	196.1	1354.3	3288.5	3857.5	2685.5	1275.2	455.1	130.9	31.9	6.9	1.3	0.2	0	13289.4
5.5	0	0	0	0	1	51	498.4	1602.9	2372.7	2008.3	1126	463.6	150.9	41	9.7	2.1	0.4	0.1	8328.1
6.5	0	0	0	0	0.2	12.6	167	690.3	1257.9	1268.6	825.9	386.8	140.8	42.2	10.9	2.5	0.5	0.1	4806.3
7.5	0	0	0	0	0	3	52.1	270.1	594.4	703.2	524.9	276.7	111.7	36.7	10.2	2.5	0.6	0.1	2586.2
8.5	0	0	0	0	0	0.7	15.4	97.9	255.9	350.6	296.9	174.6	77.6	27.7	8.4	2.2	0.5	0.1	1308.5
9.5	0	0	0	0	0	0.2	4.3	33.2	101.9	159.9	152.2	99.2	48.3	18.7	6.1	1.7	0.4	0.1	626.2
10.5	0	0	0	0	0	0	1.2	10.7	37.9	67.5	71.7	51.5	27.3	11.4	4	1.2	0.3	0.1	284.8
11.5	0	0	0	0	0	0	0.3	3.3	13.3	26.6	31.4	24.7	14.2	6.4	2.4	0.7	0.2	0.1	123.6
12.5	0	0	0	0	0	0	0.1	1	4.4	9.9	12.8	11	6.8	3.3	1.3	0.4	0.1	0	51.1
13.5	0	0	0	0	0	0	0	0.3	1.4	3.5	5	4.6	3.1	1.6	0.7	0.2	0.1	0	20.5
14.5	0	0	0	0	0	0	0	0.1	0.4	1.2	1.8	1.8	1.3	0.7	0.3	0.1	0	0	7.7
15.5	0	0	0	0	0	0	0	0	0.1	0.4	0.6	0.7	0.5	0.3	0.1	0.1	0	0	2.8
16.5	0	0	0	0	0	0	0	0	0	0.1	0.2	0.2	0.2	0.1	0.1	0	0	0	0.9
SUM:	0	0	1.3	165.4	2091.2	9279.9	19921.8	24878.8	20869.9	12898.4	6244.6	2479	836.7	247.3	65.8	15.8	3.4	0.7	100000

Figure 3　IACS Rec. 34 scatter diagram [13)]

4.　CLOSED FORMULA OF LONG-TERM PREDICTION

By approximating probability distribution of responses in short-term irregular sea state in which extreme value a exceed threshold a_j by the Rayleigh distribution, multiplying it with the occurrence probability of the short-term irregular sea state and integrating it numerically, the probability $Q[a > a_j]$ of exceeding the a_j can be expressed as equation (1). This method was proposed in the mid-1960s and have been widely used as a standard method for long-term prediction in our country [14)].

$$Q[a > a_j] = \frac{1}{2\pi}\int_0^{2\pi} \langle \iint_0^{\infty} \exp\left\{-\frac{{a_j}^2}{2[H_s\sigma(T_z,\chi)]^2}\right\} p(H_s,T_z)dH_s dT_z \rangle \, d\chi \tag{1}$$

where

a　: Extreme value

a_j　: Threshold of j mode (independent variable)

H_s　: Significant wave height

$\sigma(T_z,\chi)$　: Standard deviation per unit significant wave height in short-term irregular sea state

$p(H_s,T_z)$　: Occurrence probability density of short-term irregular sea state

T_z　: Zero-up cross mean wave period in short-term irregular sea state

χ　: Mean wave direction in short-term irregular sea state

In equation (1), the angle of encounter between the waves and the ship is assumed to be uniformly distributed because a ship sailing for long periods of time is likely to encounter waves in all directions. Assuming that the mean wave period of a short-term irregular sea state is 10 seconds, the number of waves that a ship encounters throughout its lifetime is approximately 10^8. For this reason, an exceedance probability of 10^{-8} is assumed when making long-term predictions.

In this study, the most severe short-term sea state theory proposed by Kawabe et al. [6) 14) 15)] was applied: the maximum value around the exceedance probability of 10^{-8} is dominated by the hull response in the most severe short-term sea state where the short-term parameter (standard deviation of hull response $H_s\sigma(T_z,\chi)$) of the hull response is maximized in the short-term irregular sea state constituting the long-term distribution. The maximum value of the response in the short-term irregular sea state (exactly the maximum value in the zero-up-cross mean wave period) can be approximated by the Rayleigh distribution because the spectrum of the response is a narrow band. In addition, the most severe short-term sea state that causes a long-term maximum load with an exceedance probability of 10^{-8} is defined as the short-term irregular sea state with the largest short-term parameter. Therefore, the expression in ⟨　⟩ of equation (1) is approximated as equation (2).

$$\begin{aligned}\iint_0^{\infty} \exp\left\{-\frac{{a_j}^2}{2[H_s\sigma(T_z,\chi)]^2}\right\} p(H_s,T_z)dH_s dT_z &= \sum \exp\left[-\frac{{a_j}^2}{2\left(H_{s\_i}\sigma_i\right)^2}\right] \; p\left(H_{s\_i},T_{z\_i}\right)\Delta H_s \Delta T_z \\ &\approx \exp\left[-\frac{{a_j}^2}{2\left(H_{s\_max}\sigma_{max}\right)^2}\right] \; p\left(H_{s\_max},T_{z\_max}\right)\Delta H_s \Delta T_z\end{aligned} \tag{2}$$

where

H_{s_max} : Significant wave height in most severe short-term sea state

T_{z_max} : Zero-up cross mean wave period in most severe short-term sea state

σ_{max} : Standard deviation per unit significant wave height in most severe short-term sea state

This allows the threshold of which probability of exceedance correspond to 10^{-8} to be approximated as equation (3).

$$a_j|_{Q=10^{-8}} \approx \frac{1}{2\pi}\int_0^{2\pi} \exp\left[-\frac{{a_j}^2}{2(H_{s\_max}\sigma_{max})^2}\right] \times p(H_{s\_max}, T_{z\_max})\Delta H_s \Delta T_z \ d\chi \tag{3}$$

Assuming that the duration of the short-term irregular sea state is two hours, the number of times the ship encounters the short-term irregular sea state throughout its lifetime is approximately 10^5. From this, the occurrence probability of the maximum hull response in the most severe short-term sea state is approximately taken to be 10^{-3} $(= 10^{-8}/10^{-5})$ when the occurrence probability of the maximum hull response is assumed to be 10^{-8}. By calculating the maximum expected value for 1000 waves in the short-term irregular sea state, a load equivalent to the maximum load with an exceedance probability of 10^{-8} can therefore be obtained. The maximum load with an exceedance probability of 10^{-3} can be expressed as equation (4).

$$a_j = H_{s\_max}\sigma_{max}\sqrt{2\ln 1000} \tag{4}$$

In this paper, Equation (4) is used as the base of the closed formula with an exceedance probability of 10^{-8}.

The mean wave direction in the most severe short-term sea state of heave acceleration and pitch angle were able to be obtained through the long-term prediction result of the numerical calculation and were as shown in Fig. 4. From this result, the mean wave directions in the most severe short-term sea state were set to 90 degrees and 180 degrees (heading wave) respectively.

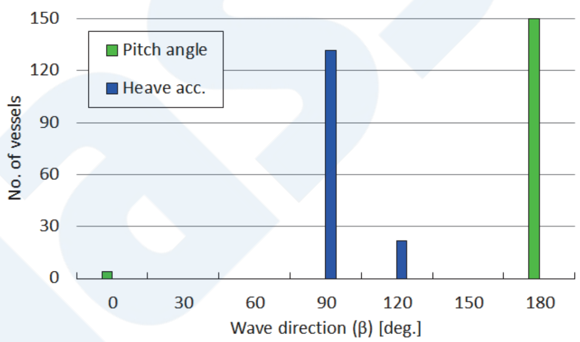


Figure 4 Mean wave direction in most severe short-term sea state

5. STANDARD DEVIATION IN THE MOST SEVERE SHORT-TERM SEA STATE

The standard deviation σ_{max} per unit significant wave height in the most severe short-term sea state is expressed as equation (5) approximately.

$$\sigma_{max} = C_1 C_2 H_{RAO\_j} \tag{5}$$

where

C_1 : Conversion coefficient for converting RAO to standard deviation per unit significant wave height

C_2 : Conversion coefficient for converting long-crested irregular wave to short-crested irregular wave

H_{RAO_j} : Maximum value of RAO of j mode

5.1 Converting to Standard Deviation per Unit Significant Wave Height

A RAO is converted to σ in order to obtain the standard deviation per unit significant wave height. Since the energy spectrum method applying the theory of linear superposition is generally used when performing short-term prediction, σ is expressed as equation (6).

$$\sigma = \sqrt{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\infty} H_{RAO\_j}(\omega, \chi - \beta)^2 S(\omega, T_z) D(\chi - \beta)\, d\omega\, d\beta} \tag{6}$$

where

$H_{RAO\_j}(\omega, \beta)$	: RAO of j mode
$S(\omega, T_z)$	: Pierson-Moskowitz type wave spectrum per unit significant wave height
$D(\beta)$	: Directional distribution function
ω	: Wave frequency
β	: Direction of wave component

The term excluding the directional distribution function in equation (6) was conveniently divided by the maximum value of RAO, and equation (7) is given for the conversion coefficient C_1. Note that the sea state at this time is the most severe short-term sea state.

$$C_1 = \frac{\sqrt{\int_0^{\infty} H_{RAO\_j}(\omega, \chi_{max})^2 S\left(\omega, T_{z\_max}\right) d\omega}}{H_{RAO\_j}\left(\omega_{RAOpeak}, \chi_{max}\right)} \tag{7}$$

where

$\omega_{RAOpeak}$	: Wave frequency when RAO is maximized
χ_{max}	: Mean wave direction in most severe short-term sea state

C_1 is, simply put, a coefficient for converting the RAO for a regular wave into standard deviation value in consideration of the wave spectrum shape of irregular wave. Since C_1 will differ in size depending upon hull response, it must be determined for each hull response.

Froude-krylov force is the main component of the hydrodynamic force acting on the hull required to determine the hull motion. The hydrodynamic forces affecting vertical motion, such as heave and pitch, are values integrated with respect to n_z, and thus greatly contribute to the hull shape in the z-direction projection plane area, that is, the water plane area. From this, it was considered that the C_1 values for heave acceleration and pitch angle can be expressed by the equation of the water plane area LBC_w, and determined as in equation (8).

Heave acceleration	$C_1 = 0.03(LBC_w)^{0.18}$	(8)
Pitch angle	$C_1 = 0.12(LBC_w)^{0.05}$	

Figure 5 shows a comparison of C_1 values obtained from equation (8) and numerical calculation results. Both C_1 are mostly as expected, indicating that they were able to set relatively high-precision formulae.

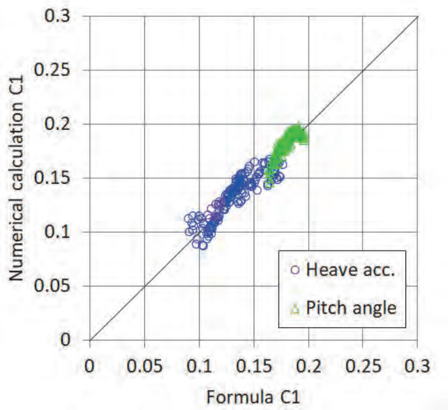


Figure 5 Comparison of equation (8) and numerical calculation C_1 values

The wave spectrum of the irregular wave used in the calculation of C_1 is the Pierson-Moskowitz type recommended by IACS Rec. 34 [13] as shown in equation (9). This wave spectrum is determined by the significant wave height and the zero-up cross mean wave period of the North Atlantic. Note that this Pierson-Moskowitz type wave spectrum equation is conveniently divided by $H_s{}^2$.

$$S(\omega, T_z) = \frac{1}{4\pi}\left(\frac{2\pi}{T_z}\right)^4 \omega^{-5} \exp\left[-\frac{1}{\pi}\left(\frac{2\pi}{T_z}\right)^4 \omega^{-4}\right] \tag{9}$$

5.2 Converting to Standard Deviation per Unit Significant Wave Height

Although the variance σ^2 is obtained from the integral of the hull response spectrum, this value is that of a long-crested irregular wave. The wave fields encountered under actual sea conditions are rarely long-crested irregular waves but rather short-crested irregular waves in which irregular waves arriving from various directions overlap. For a more accurate statistical prediction, the σ^2 needs to be converted to value for short-created irregular waves. That is to say, a short-crested irregular wave having a set of wave components of different frequencies and a planar spread is approximately represented by using a directional distribution function $D(\beta)$ as shown in equation (10) [16].

$$D(\beta) = \frac{(2n)!}{\pi(2n-1)!}[\cos(\chi-\beta)]^{2n} \qquad \left(-\frac{\pi}{2} \le \beta \le \frac{\pi}{2}\right) \tag{10}$$

In the short-term prediction of a hull response of the Fukuda method [17], equation (11) where n=1 is often used.

$$D(\beta) = \frac{2}{\pi}[\cos(\chi-\beta)]^2 \qquad \left(-\frac{\pi}{2} \le \beta \le \frac{\pi}{2}\right) \tag{11}$$

Assuming that the standard deviation of the short-crested irregular wave is σ_{short} and the standard deviation of the long-crested irregular wave is σ_{long}, σ_{short} can be expressed as equation (12).

$$\sigma_{short} = \sqrt{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sigma_{long}{}^2 D(\chi-\beta)\, d\beta} \tag{12}$$

where

σ_{short} Standard deviation in short-crested irregular waves

σ_{long} Standard deviation in long-crested irregular waves

The coefficient C_2 that converts long-crested irregular wave to short-crested irregular wave in most severe short-term sea state was defined by equation (13).

$$C_2 = \frac{\sigma_{short}\left(T_{z\_max}, \chi_{max}\right)}{\sigma_{long}\left(T_{z\_max}, \chi_{max}\right)} \tag{13}$$

Since the ratio of hull response for each wave direction is basically the same regardless of the size of the ship when considering a hull motion, it can be inferred that C_2 is almost constant. From numerical calculation results, it can also be confirmed that C_2 is plotted as shown in Fig. 6, and its tendency is almost as estimated.

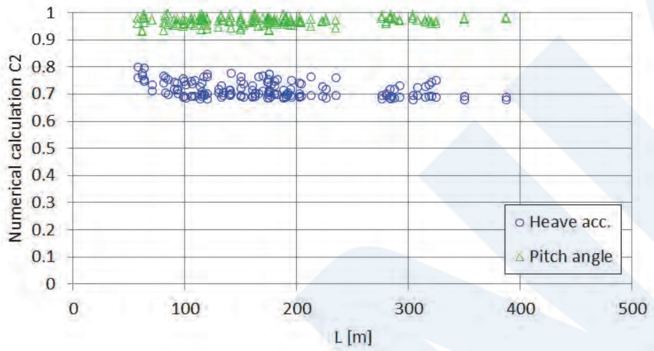


Figure 6 Numerical calculation C_2 values

From Fig. 6, the C_2 was expressed as shown in equation (14).

$$\begin{array}{ll} \text{Heave acceleration} & C_2 = 0.72 \\ \text{Pitch angle} & C_2 = 0.97 \end{array} \tag{14}$$

6. SIGNIFICANT WAVE HEIGHT IN THE MOST SEVERE SHORT-TERM SEA STATE

The probability model [18] used in this study is a joint probability distribution of the conditional probability distribution of the wave period with respect to the significant wave height (log normal distribution) and the marginal probability distribution of the significant wave height (Weibull distribution) as shown in equation (15).

$$p(T_z|H_s)p(H_s) = \frac{1}{T_z\sqrt{2\pi\sigma_T(H_s)}}\exp\left\{-\frac{[ln\, T_z - m_T(H_s)]^2}{2\sigma_T{}^2(H_s)}\right\} \times \frac{\beta(H_s-\gamma)^{\beta-1}}{\alpha^\beta}\exp\left[-\left(\frac{H_s-\gamma}{\alpha}\right)^\beta\right] \tag{15}$$

where

α : Scale parameter

β : Shape parameter

γ : Threshold

with $m_T(H_s)$ and $\sigma_T{}^2(H_s)$ obtained as shown in equation (16).

$$\begin{aligned} m_T(H_s) &= E(\ln T_z(H_s)) \\ \sigma_T{}^2(H_s) &= Var(\ln T_z(H_s)) \end{aligned} \tag{16}$$

The scatter diagram given in IACS Rec. 34 [13] shown in Fig. 3 was used and the occurrence probability of the short-term irregular sea state of equation (15) was 10^{-5}, the significant wave height at that probability can be determined by polynomial approximation as shown in equation (17).

$$H_{s\_max} = -0.21{T_{z\_max}}^2 + 5.07T_{z\_max} - 15.7 \tag{17}$$

Equation (17) is plotted as Fig. 7. From Fig. 7, it can be seen that equation (17) represents the significant wave height of a sea state with an extremely low frequency of occurrence. Incidentally, the short-term irregular sea state in which the joint probability in the scatter diagram is 10^{-5} is generally circular distribution (there is a case where two short-term irregular sea states occur in a zero-up cross mean wave period). However, since the higher significant wave height of the wave heights is generally used in ship design, the approximate expression of the significant wave height is made to be a second order polynomial. The range of monotonically increasing and monotonically decreasing the significant wave height is limited, because the short-term irregular sea state in which the joint probability is 10^{-5} becomes the circular distribution. Therefore, we consider about 17.0 s to be an appropriate upper limit for the zero-up cross average wave period which is a variable of equation (17).

Hs/Tz	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	14.5	15.5	16.5	17.5	18.5	SUM
0.5	0.0	0.0	1.3	133.7	865.6	1186.0	634.2	186.3	36.9	5.6	0.7	0.1	0.0	0.0	0.0	0.0	0.0	0.0	3050
1.5	0.0	0.0	0.0	29.3	986.0	4976.0	7738.0	5569.7	2375.7	703.5	160.7	30.5	5.1	0.8	0.1	0.0	0.0	0.0	22575
2.5	0.0	0.0	0.0	2.2	197.5	2158.8	6230.0	7449.5	4860.4	2066.0	644.5	160.2	33.7	6.3	1.1	0.2	0.0	0.0	23810
3.5	0.0	0.0	0.0	0.2	34.9	695.5	3226.5	5675.0	5099.1	2838.0	1114.1	337.7	84.3	18.2	3.5	0.6	0.1	0.0	19128
4.5	0.0	0.0	0.0	0.0	6.0	196.1	1354.3	3288.5	3857.5	2685.5	1275.2	455.1	130.9	31.9	6.9	1.3	0.2	0.0	13289
5.5	0.0	0.0	0.0	0.0	1.0	51.0	498.4	1602.9	2372.7	2008.3	1126.0	463.6	150.9	41.0	9.7	2.1	0.4	0.1	8328
6.5	0.0	0.0	0.0	0.0	0.2	12.6	167.0	690.3	1257.9	1268.6	825.9	386.8	140.8	42.2	10.9	2.5	0.5	0.1	4806
7.5	0.0	0.0	0.0	0.0	0.0	3.0	52.1	270.1	594.4	703.2	524.9	276.7	111.7	36.7	10.2	2.5	0.6	0.1	2586
8.5	0.0	0.0	0.0	0.0	0.0	0.7	15.4	97.9	255.9	350.6	296.9	174.6	77.6	27.7	8.4	2.2	0.5	0.1	1309
9.5	0.0	0.0	0.0	0.0	0.0	0.2	4.3	33.2	101.9	159.9	152.2	99.2	48.3	18.7	6.1	1.7	0.4	0.1	626
10.5	0.0	0.0	0.0	0.0	0.0	0.0	1.2	10.7	37.9	67.5	71.7	51.5	27.3	11.4	4.0	1.2	0.3	0.1	285
11.5	0.0	0.0	0.0	0.0	0.0	0.0	0.3	3.3	13.3	26.6	31.4	24.7	14.2	6.4	2.4	0.7	0.2	0.1	124
12.5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	1.0	4.4	9.9	12.8	11.0	6.8	3.3	1.3	0.4	0.1	0.0	51
13.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	1.4	3.5	5.0	4.6	3.1	1.6	0.7	0.2	0.1	0.0	21
14.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	1.2	1.8	1.8	1.3	0.7	0.3	0.1	0.0	0.0	8
15.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	0.6	0.7	0.5	0.3	0.1	0.1	0.0	0.0	3
16.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.2	0.2	0.1	0.1	0.0	0.0	0.0	1
SUM:	0	0	1	165	2091	9280	19922	24879	20870	12898	6245	2479	837	247	66	16	3	1	100000

Figure 7　Plotted equation (17) on IACS Rec. 34 scatter diagram [13]

7. WAVE PERIOD IN THE MOST SEVERE SHORT-TERM SEA STATE

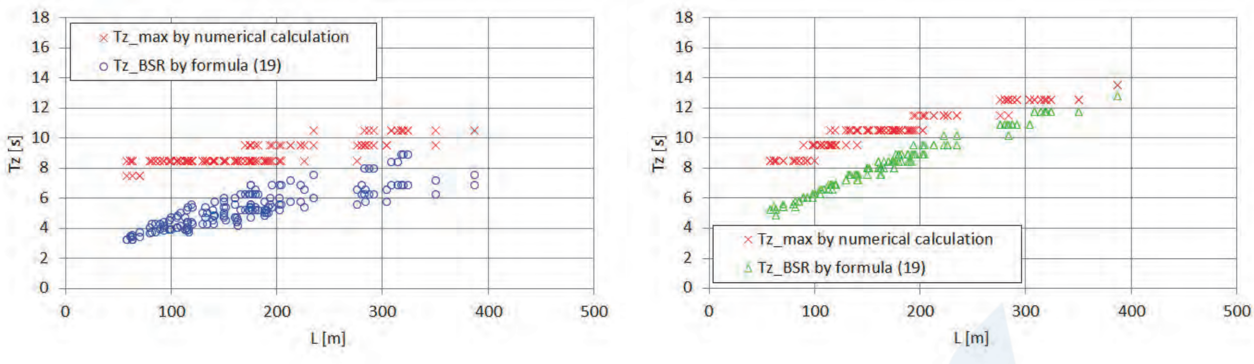
In general, wave period of the regular wave at which RAO is maximized can be expressed as in equation (18) by using wave frequency $\omega_{RAOpeak}$ at which RAO is maximized.

$$T_{RAOpeak} = \frac{2\pi}{\omega_{RAOpeak}} \tag{18}$$

Assuming a response spectrum has narrow-band characteristics, the standard deviation value in the short-term irregular sea state is maximized when the peaks of both the wave spectrum with a mean wave period and the RAO are superimposed at the same wave frequency. This is called "broad sense of resonance" [14) 15)]. According to the assumption, the zero-up cross mean wave period of broad sense of resonance [14) 15)] T_{z_BSR} can be expressed as following equation.

$$T_{z\_BSR} = 0.71T_{RAOpeak} \tag{19}$$

The coefficient 0.71 of equation (19) means the converting factor from the peak period of wave spectrum to zero-up cross mean wave period of Pierson-Moskowitz type wave spectrum. Figure 8 shows a comparison between the zero-up cross mean wave period of broad sense of resonance T_{z_BSR} by equation (19) and the zero-up cross mean wave period for the most severe short-term sea state T_{z_max} obtained by the numerical calculation.



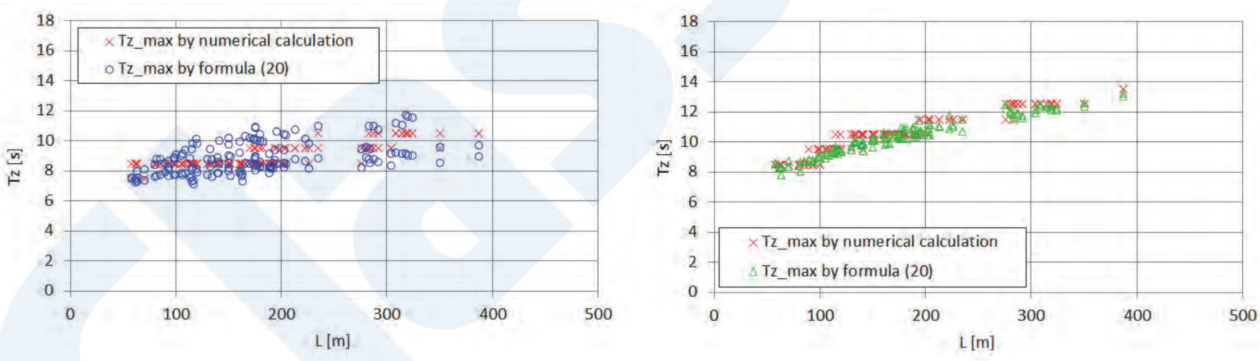
(A) Heave Acceleration (B) Pitch Angle

Figure 8 Wave period (equation (19) and numerical calculation results)

From Fig. 8, it can be seen that T_{z_max} tends to be longer than T_{z_BSR}. This is because, the significant wave height is considered in T_{z_max} but not in T_{z_BSR}. In order to convert T_{z_BSR} into T_{z_max}, the coefficient is determined by the relationship shown in Fig. 8 is expressed by using the water plane area LBC_w and the wave period in the most severe short-term sea state is expressed as in equation (20).

$$\begin{aligned} &\text{Heave acceleration} \qquad T_{z\_max} = 6.20(LBC_w)^{-0.16}T_{z\_BSR} \\ &\text{Pitch angle} \qquad T_{z\_max} = 3.67(LBC_w)^{-0.13}T_{z\_BSR} \end{aligned} \tag{20}$$

Figure 9 shows a comparison between equation (20) and the zero-up cross mean wave period for the most severe short-term sea state T_{z_max} obtained by the numerical calculation. As can be seen, the zero-up cross mean wave period in the most severe short-term sea state can be estimated with high accuracy by the conversion.



(A) Heave Acceleration (B) Pitch Angle

Figure 9 Wave period (equation (20) and numerical calculation results)

8. WAVE FREQUENCY

As can be seen from equation (18), (19) and (20), in order to obtain T_{z_max}, the wave frequency $\omega_{RAOpeak}$ at which RAO is maximized is required. The RAO of the heave acceleration reaches its maximum value when the peaks of both the wave spectrum and the RAO are superimposed at the same wave frequency. When the RAO of the heave acceleration reaches its maximum value in a wave direction of 90 degrees, this maximum value becomes extremely large, whereas the sum $-\omega_e{}^2(M_{33}+A_{33})+C_{33}$ of the inertia term (mass and heave added mass) and the buoyancy term (heave restoring force coefficient) tends to be almost 0. Therefore, the $\omega_{RAOpeak}$ of heave acceleration is the wave frequency when the sum of the inertia term and the buoyancy term becomes 0, and is expressed as shown in equation (21). Equation (21) is based on the simplified formula of hydrodynamic coefficient by Matsui et al. [9]. As shown in Fig. 10, a comparison of the $\omega_{RAOpeak}$ values for heave acceleration obtained from equation (21) and from numerical calculation results indicates that the accuracy of the equation (21) is high.

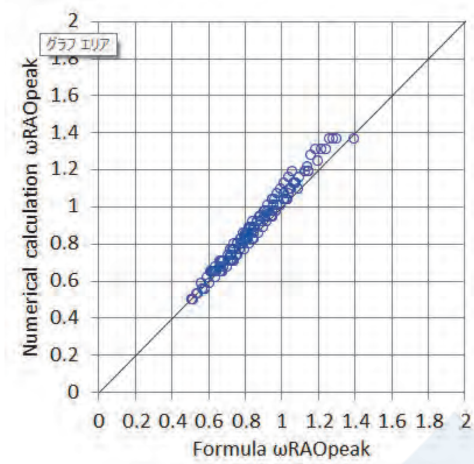


Figure 10 Comparison of $\omega_{RAOpeak}$ of heave acceleration between equation (21) and numerical calculation

On the other hand, when the wave direction is 180 degrees, the RAO of the pitch angle does not reach its maximum value when the peaks of both the wave spectrum with a mean wave period and the RAO are superimposed at the same wave frequency. The method used to obtain the $\omega_{RAOpeak}$ value for heave acceleration cannot be used. Therefore, based on the qualitative idea that $\omega_{RAOpeak}$ of the pitch angle does not depend on the ship length, when the $\omega_{RAOpeak}$ obtained from numerical calculation is non-dimensionalized by the ship length, the non-dimensional values of the $\omega_{RAOpeak}$ were expressed as Fig. 11.

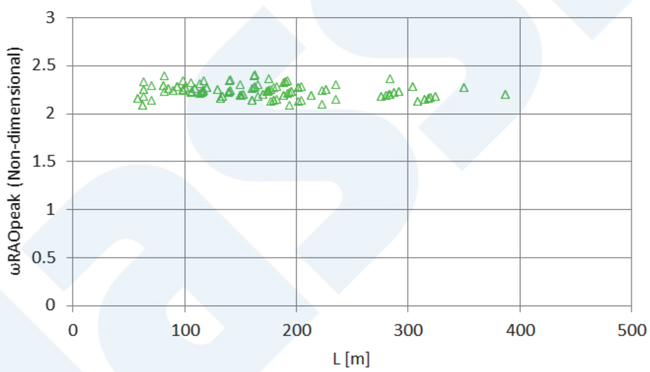


Figure 11 Non-dimensional $\omega_{RAOpeak}$ of pitch angle of numerical calculation

From Fig. 11, it was able to be seen that the non-dimensional $\omega_{RAOpeak}$ of the pitch angle is almost constant regardless of the ship length. From this reason, the non-dimensional $\omega_{RAOpeak}$ was determined as 2.23, and the $\omega_{RAOpeak}$ was determined as in equation (21).

Heave acceleration

$$\omega_{RAOpeak} = \sqrt{\frac{gC_w}{dC_b + 0.108\pi B\dfrac{2C_w{}^2}{C_w + 1}}} \tag{21}$$

Pitch angle

$$\omega_{RAOpeak} = 2.23\sqrt{\frac{g}{L}}$$

As shown in Fig. 12, the $\omega_{RAOpeak}$ values for pitch angle obtained from equation (21) and from numerical calculation results were compared. The $\omega_{RAOpeak}$ was able to set relatively high-precision formula.

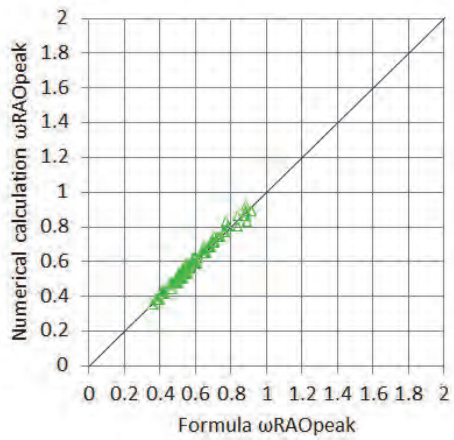
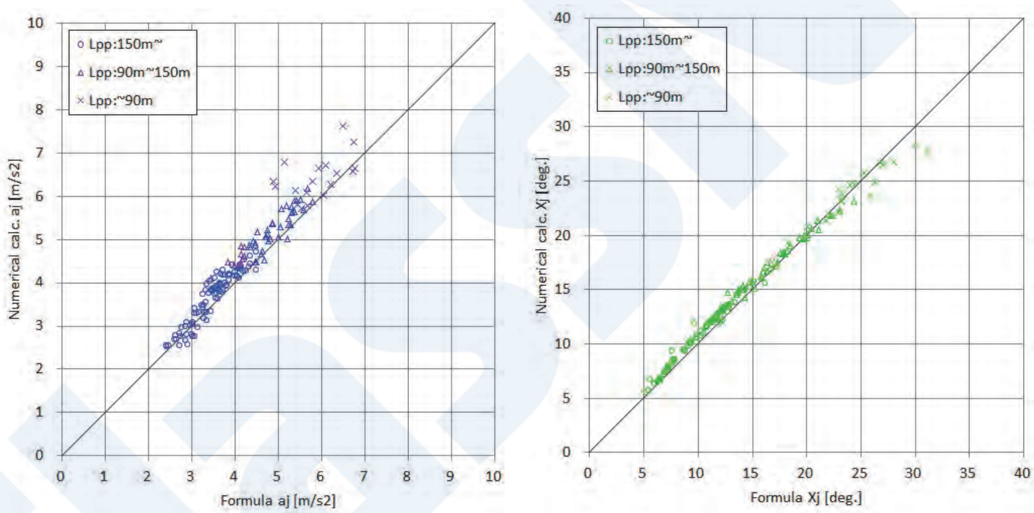


Figure 12 Comparison of $\omega_{RAOpeak}$ of pitch angle between equation (21) and numerical calculation

9. COMPARISON OF CLOSED FORMULA AND NUMERICAL CALCULATION

Figure 13 compares the long-term predicted values with an exceedance probability of 10^{-8} for the heave acceleration and pitch angle values obtained from equation (4) to the numerical calculation results. As can be seen, the accuracy of the closed formula is high.



(A) Heave Acceleration (B) Pitch Angle

Figure 13 Long-term predicted values of exceedance probability of 10^{-8} (equation (4) and numerical calculation results)

10. CONCLUSION

Highly accurate closed formulae of long-term prediction for heave acceleration and pitch angle were developed based on linear theory. The components of this proposed formula were introduced in Fig. 1. The components were briefly described as follows.

- Assuming the standard deviation in most severe short-term sea state of hull response has correlation with the peak RAO value, the value was used in order to formulate a long-term prediction. In this paper, RAO values obtained from numerical calculations in order to confirm proposed formulae was used; they were not formulated.
- Coefficients to convert from the peak value of RAO into standard deviation were formulated for each response using the water plane area LBC_w from qualitative hull motion phenomenon. (Sec. 5.1)
- Coefficients to convert from long-crested irregular waves into short-crested irregular waves were constant values for each

response. (Sec. 5.2)

- Standard deviation per unit significant wave height in the most severe short-term sea state was formulated by multiplying the coefficients of Sec. 5.1 and 5.2. (Sec. 5)
- Wave frequency of heave acceleration at which RAO is maximized was formulated from values when both the sum of the inertia term and the buoyancy term become 0. Wave frequency of pitch angle was formulated from the non-dimensional $\omega_{RAOpeak}$ based on the qualitative idea that $\omega_{RAOpeak}$ of the pitch angle does not depend on the ship length. (Sec. 8)
- Zero-up cross mean wave periods in the most severe short-term sea state were formulated based on the extremum of wave spectrum, so-called “broad sense of resonance” [14) 15)], and correction factors were determined by using the water plane area LBC_w for heave acceleration and pitch angle respectively. (Sec. 7)
- Significant wave height of an exceedance probability of 10^{-5} was formulated by using a joint probability distribution as a probability model for significant wave height and zero-up cross mean wave period. (Sec. 6)
- Maximum load with an exceedance probability of 10^{-8} was formulated by using most severe short-term sea state theory6). (Sec. 4)

As described above, the closed formulae presented in this paper were based upon RAO values obtained from numerical calculations. It is planned to use the closed formulae of the RAO developed based upon the theory of seakeeping analysis by author et al. Moreover, there are plans to develop closed formulae for lateral motions and bending moments following the same approach used in this paper.

ACKNOWLEDGMENT

The authors wish to express their profound gratitude to Dr. Toichi Fukasawa of the National Maritime Research Institute (NMRI) for their appropriate advice in conducting this research and preparing the paper, and to Dr. Hiroshi Kawabe of Nippon Kaiji Kyokai (ClassNK), Mr. Tatsuya Akamatsu of Nippon Yusen Kaisha (NYK Line), Mr. Tomoki Omiya, Mr. Kumpei Wakida and Mr. Kazushi Kuzuya of Mitsui O.S.K. Lines, Ltd. and Mr. Junya Matsuwaki of Imabari Shipbuilding Co., LTD. for their cooperation in the data analysis in this research.

REFERENCES

1) K. Sugimoto, Y. Fukumoto, J. Matsuwaki, T. Akamatsu, S. Ashida, K. Onishi, H. Houtani, M. Oka, H. Kawabe, K. Ishibashi, Non Linear Ef-fect on Wave-induced Loads for Hull Structural Design, Proceeding of 39th International Con-ference on Offshore Mechanics and Arctic En-gineering (OMAE 2020), OMAE ASME 2020
2) International Association of Classification So-cieties, Common Structural Rules for Bulk Carriers and Oil Tankers, 2019
3) ClassNK, Rules for the Survey and Construc-tion of Steel Ships Part C, 2019
4) A. Shinkai, Estimating the Design Values of Vertical Bending Moment Induced on the Ship Hull in Waves, Journal of the Society of Naval Architects of Japan, Vol. 138, pp. 295-304, 1975 (in Japanese)
5) J. Fukuda, R. Nagamoto, O. Tsukamoto, A. Shinkai, Estimating the Design Values of Hor-izontal Wave Shearing Force Induced on the Ship Hull in Waves, Journal of the Society of Naval Architects of Japan, Vol. 139, pp. 166-173, 1976 (in Japanese)
6) H. Kawabe, M. Morikawa, K. Shibazaki, Simple Estimation Method for Long-term Distribution of Wave Induced Load based on the Severest Wave Condition Approach, Journal of the So-ciety of Naval Architects of Japan, Vol. 189, pp. 193-200, 2001 (in Japanese)
7) T. Shigemi, T. Zhu, Studies on the Practical Estimation Method of the Design Loads for Primary Structural Members of Tankers - 1st Report, Design Sea States -, Journal of the So-ciety of Naval Architects of Japan, Vol. 191, pp. 195-207, 2002 (in Japanese)
8) J. J. Jensen, A. E. Mansour, A. S. Olsen: Es-timation of ship motions using closed-form expressions, Ocean Engineering, Vol. 31, pp. 61-85, 2004
9) S. Matsui, K. Shinomoto, K. Sugimoto, S. Ashida, Development of Closed Formula of Ship Response in Wave – 2nd

report: Hydro-dynamic Forces Related to Vertical Motion-, Conference proceedings, the Japan Society of Naval Architects and Ocean Engineers, Vol 28, 2019 (in Japanese)

10) M. Oka, T. Ando, C. Ma, T. Yoshida, N. Izumi, N. Matsubara, A. Usami, Y. Kidoura, H. Ka-wabe, Numerical Calculation Procedure of Global Ship Motion of LNG Carrier Considering LNG Liquid Dynamic Effect in Wave Condition, Journal of the Japan Society of Naval Archi-tects and Ocean Engineers, Vol. 26, pp. 103-118, 2017 (in Japanese)

11) A. D. Papanikolaou, T. E. Schellin, A Three Dimensional Panel Method for Motions and Loads of Ships with Forward speed, Ship technology research, Vol.39, pp. 145-156, 1992

12) International Association of Classification So-cieties, Technical Background Documents for CSR, 2018

13) International Association of Classification So-cieties, Recommendation No. 34 Standard Wave Data, 2001

14) H. Kawabe, S. Hibi, H. Tanaka, K. Shibazaki, H. Sasajima, Contribution of Supposed Wave Condition on Long-term Distribution of Wave Induced Load (1st Report Relation between The Maximum Wave Induced Load and Sup-posed Wave Condition), Journal of the Society of Naval Architects of Japan, Vol. 186, pp. 319-339, 1999 (in Japanese)

15) H. Kawabe, Contribution of Supposed Wave Condition on The Long-term Distribution of A Wave Induced Load, Journal of Marine Science and Technology, Vol. 6, pp. 135-147, 2002

16) S. Takezawa, K. Kobayashi, On the Motion Responses of Offshore Floating Structures in Directional Spectra Waves -The 1st-order re-sponses-, Journal of the Society of Naval Ar-chitects of Japan, Vol. 165, pp. 141-152, 1989 (in Japanese)

17) J. Fukuda, Statistic Prediction of Ship Re-sponse, Society of Naval Architects of Japan, First Symposium on Ship Manoeuvrability, pp. 99-119, 1969 (in Japanese)

18) A. Shinkai, S. Wan, The Statistical Character-istics of Wave Data and Long-Term Predictions of the Ship Response, Transactions of the West-Japan Society of Naval Architects, Vol. 89, pp. 223-231, 1995 (in Japanese)