KC#529

Derivation of the normal stress terms in the formulae for p_{zi} CH6, Sec3, 4.2.2
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The formulae for the nominal pressures p_{zx} and p_{zy} contain the influence of normal stresses.

$$p_{zx} = \frac{t_a}{b}\left(\sigma_{xl}\left(\frac{\pi \cdot b}{a}\right)^2 + 2 \cdot c_y \cdot \sigma_y + \sqrt{2}\tau_1\right)$$

$$p_{zy} = \frac{t_a}{a}\left(2 \cdot c_x \cdot \sigma_{xl} + \sigma_y\left(\frac{\pi \cdot a}{n \cdot b}\right)^2\left(1 + \frac{A_y}{a \cdot t_a}\right) + \sqrt{2}\tau_1\right)$$

with $\sigma_{xl} = \sigma_x\left(1 + \frac{A_x}{b \cdot t_a}\right)$

The axial acting stresses can be calculated according the TB document ***New Req. of GL for Proof of Buckling Strength (inkl DIN18800).pdf***

$$M_\ell = \frac{q\ell^2}{\pi^2} = F_{Ki}\frac{\frac{\pi^2}{\ell^2}F \cdot w_o}{\frac{\pi^2}{\ell^2}F_{Ki} - \frac{\pi^2}{\ell^2}F} = F_{Ki}\frac{p_{zl} \cdot w_o}{c_f - p_{zl}}$$

where:

p_{zl} = load caused by the longitudinal force

p_{zl} = $\frac{\pi^2}{\ell^2} \cdot F = \frac{\pi^2}{\ell^2}\sigma_a \cdot A$

The following derivation shows the equivalence of the formula for p_{zx} and the first term of the formula for p_{zx}.

$$\frac{t_a}{b}\left(\sigma_{xl}\left(\frac{\pi \cdot b}{a}\right)^2\right) = \sigma_x\left(\frac{t_a\pi^2b^2}{a^2b} + \frac{t_a\pi^2b^2A_x}{a^2b \cdot bt_a}\right)$$

$$= \sigma_x\frac{\pi^2}{a^2}\left(t_ab + A_x\right) = \sigma_x\frac{\pi^2}{a^2}A$$

This derivation shows in addition, that the stress σ_x in the formulae for p_{zx} and p_{zy} is the axial stress, acting in the stiffener.

$\sigma_x = \sigma_a$ with $\sigma_a = \sigma_x^* - 0.3 \cdot \sigma_y^*$ where σ_x^* and σ_y^* are FE-stresses.

In case of a transverse stiffener, the stress of the second term with σ_y can be derived accordingly.

In the two formulae for p_{zx} and p_{zy} are different assumptions.

- Longitudinal stiffeners
  - are not supported by intermediate transverse stiffeners.
  - act always as secondary members
- Transverse stiffeners
  - are classified in two kind of stiffeners
    a) small transverse stiffener (secondary member), located between two longitudinals
    b) transverse stiffener with higher rigidity than the longitudinal stiffener, which is supported by intermediate longitudinal stiffeners and partially constrained at the ends (primary member)

    The factor c_s characterises the support condition at the ends of the transverse stiffener in line with the categorisation. A secondary (transverse stiffener) is simply supported, a primary stiffener is partially constrained.

In case a) the stress acting normal to the stiffener is σ_x^*. No intermediate longitudinals have to be taken into account, so A_x is zero and the term $\sigma_{xl} = \sigma_x^*$. In this case is $n \neq 1$.

In case b) the intermediate longitudinal stifferners provide an additional support for the transverse stiffener and shift the acting normal stress out of the plate plane. It is obvious that the normal stress has to be reduced.

$$\frac{t_a}{a}\left(2c_x\sigma_{xl}\right) = \sigma_x\left(\frac{2t_a c_x}{a} + \frac{2t_a c_x A_x}{abt_a}\right)$$
$$= 2c_x\sigma_x\left(\frac{t_a}{a} + \frac{A_x}{ab}\right)$$

Both terms $\left(\frac{t_a}{a}\right)$ and $\left(\frac{A_x}{ab}\right)$ are much more smaller than 1 and represent the reduction factors for the normal acting stress σ_x with $\sigma_x = \sigma_a$.