KC#291

ClassNK

Stress Concentration Factor for Knuckle Part
(Stool Structure, Bilge Hopper Structure, etc.)

1. Basic SCF

Stress condition around the knuckle part is generally depending on the angle of knuckle part and on the distance from knuckle point. According to the Williams' solution (R.William and Soutas-Little, "Elasticity", Dover publishing, 1998), stress on the plate surface around knuckle part is given by the equation below:

$$\sigma_r = \frac{A'}{r^{\lambda-2}} \tag{1}$$

where r ; Distance from the knuckle point to the evaluated point

$\lambda-2$; Parameter which shows the degree of the effect of singular point depending on the angle to the horizontal of sloped plate, θ

A' ; Coefficient

Since the value of $\lambda-2$ is depending on θ, the equation which gives the approximate to a strict solution of $\lambda-2$ was examined. Then the simplified approximate equation is obtained as below. Table 1 shows the comparison between the strict solution and the approximation.

$$\lambda-2 = 0.2 + 0.0028\cdot\theta \tag{2}$$

Table 1 Comparison between the strict solution and the approximation

θ (deg.)	$\lambda-2$	$0.2+0.0028\cdot\theta$
90	0.4556	0.4520
75	0.4261	0.4100
60	0.3840	0.3680
45	0.3265	0.3260
30	0.2480	0.2840
20	0.1813	0.2560

According to the equation (1), the value σ_r is asymptotic in a certain value when r becomes around 300mm to 500mm. If an asymptotic value is assumed to be a nominal stress, the stress concentration factor can be defined as below.

$$\sigma_r = \frac{A'}{r^{\lambda-2}} = \frac{A}{r^{\lambda-2}}\cdot\sigma_n = K_0\cdot\sigma_n \qquad \left[\, K_0 \equiv \frac{A}{r^{\lambda-2}} \,\right] \tag{3}$$

When the stress around the knuckle part is evaluated by the FE analysis, coefficient A is expressed as below.

$$A(r) = \frac{\sigma_r}{\sigma_n} r^{\lambda-2} = \frac{\sigma_{FEM(r)}}{\sigma_n} r^{\lambda-2} \tag{4}$$

Although the coefficient A is originally depending on the distance from the knuckle point, the coefficient A is evaluated as the mean value around the knuckle part to generalize the stress concentration factor. In order to evaluate the value of coefficient A, FE analyses of stool and bilge hopper structures of bulk carriers and bilge knuckle structures of tankers were made. The obtained results are shown in Fig.1. Since the value of coefficient A shows strong dependency on the angle, the approximated equation of A with the angle is obtained as below.

$$\overline{A} = 0.14\,\theta\cdot(1.15 - 0.0033\,\theta) \tag{5}$$

On the other hand, from an engineering viewpoint, the stress concentration factor is defined as the ratio of the hot spot stress evaluated by the FE analysis to the nominal stress as below.

$$K_0 \equiv \frac{\sigma_{hot\ spot}}{\sigma_n} \tag{6}$$

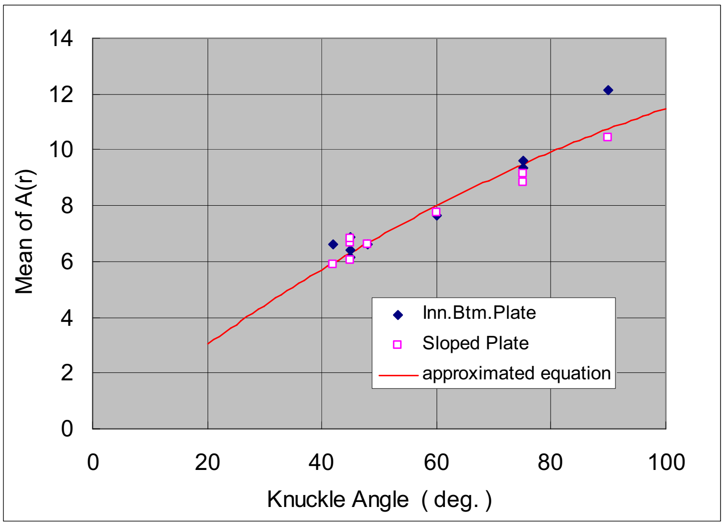


Fig. 1 Evaluated Value of Coefficient '*A*'

In general, the hot spot stress is obtained by the linear extrapolation based on the stresses at the points where 0.5t and 1.5t part from the hot spot position. Table 2 shows the evaluated stress at the points 0.5t and 1.5t part from the hot spot position and the extrapolated hot spot stress. According to the results shown in Table 2, the calculated stress according to the approximated equation at a point 0.5t part from the knuckle point is almost same as the extrapolated stress based on the FE analysis.

Therefore, the stress concentration factor for the knuckle part can be defined as the equation below.

$$K_0 \equiv \frac{A}{r^{\lambda-2}} = \frac{0.14\theta \cdot (1.15 - 0.0033\theta)}{(0.5 \cdot t)^{(0.2+0.0028\cdot\theta)}} \qquad [\sigma_{hot} = K_0 \times \sigma_n] \tag{7}$$

Table 2 Comparison of the hot spot stress

θ (deg.)	*0.5 t*		*1.5 t*		hot spot		(e) / (b)
	FEM(a)	formula(b)	FEM(c)	formula(d)	FEM(e)	formula(f)	
45	18.6	18.9	14.3	13.2	20.8	21.8	1.10
45	20.6	24.7	16.8	17.6	22.5	28.2	0.91
45	12.4	13.4	9.5	9.4	13.9	15.4	1.03
45	14.4	15.2	11.9	10.6	15.7	17.5	1.03
60	12.0	13.8	9.1	9.2	13.5	16.1	0.97
60	14.6	19.3	12.2	12.9	15.8	22.5	0.82
75	14.0	15.4	10.3	9.8	15.9	18.2	1.03
75	16.8	23.2	13.4	14.8	18.5	27.4	0.80
75	7.9	9.1	5.8	5.8	9.0	10.8	0.98
75	10.2	13.6	8.3	8.7	11.2	16.1	0.82
90	9.7	10.1	7.0	6.2	11.1	12.1	1.09
90	12.1	16.5	9.4	10.0	13.5	19.8	0.82
45	24.2	25.7	19.9	18.0	26.4	29.6	1.03
45	20.7	24.2	17.8	16.9	22.2	27.9	0.92
48	19.6	20.9	15.4	14.5	21.7	24.1	1.04
48	15.9	18.0	13.9	12.5	16.9	20.8	0.94
42	9.9	8.0	7.5	5.6	11.1	9.2	1.39
42	10.7	11.7	8.8	8.3	11.7	13.4	1.00

2. Correction coefficient for SCF

2.1 Influence Factors on Stress Concentration

When the stress concentration factor of an actual structure is evaluated, it is necessary to correct the basic stress concentration factor because this value is the one evaluated based on the solution to two dimensional plane problem. The following correction coefficients will be necessary if the difference between an actual structure and the idealization model, which is the base of K_0.

$$K_t = K_0 \times K_1 \times K_2 \times K_3 \times K_4$$

where K_0 ; SCF depending on the dimensions of the considered structure (Eqn.(7) & Table 2)
K_1 ; Correction factor depending on the plate bending process
K_2 ; Correction factor depending on the thickness increment of the web plate
K_3 ; Correction factor depending on the insertion of horizontal gusset or longitudinal rib
K_4 ; Correction factor depending on the insertion of transverse rib

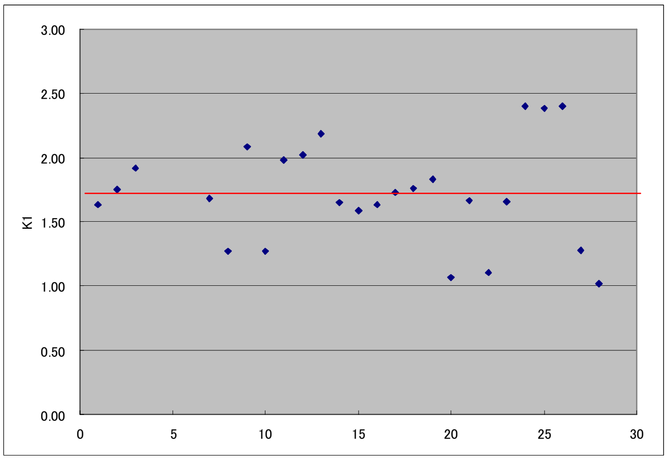
In order to evaluate the above mentioned correction factors, a number of FE analyses were made as shown in Table 3. According to the results of FE analyses, following values were evaluated as the correction factors.

Table 3 FEM Analyses

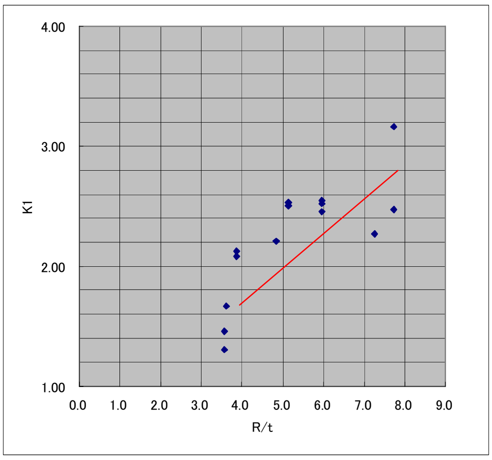
part of structural	knuckle type	longitudinal rib	transverse rib
VLCC Lower knuckle	R=120 - 800	none / attached	none / attached
	weld type	none / attached	none / attached
Bulk Carrier Bilge knuckle	R=60 - 120	none / attached	none / attached
	weld type	none / attached	none / attached
VLCC Upper knuckle	R=120 - 800	none	none
	weld type	none	none

2.1 Correction Factor

Figure 2 shows the evaluated value of correction factor K_1 according to the FE analyses for weld type structures and bend type structures. The correction factor K_1 for weld type structure does not depend on the plate thickness but K_1 for bend type structure depends on the radius of bend part and the plate thickness. According to the results, the correction factors K_1 for weld type and bend type could be obtained as below:



(a) weld type



(b) bend type

Fig. 2 Evaluated Value of Correction Factor K1

$$K_{1,weld} = 1.7$$

$$K_{1,bend} = \begin{cases} 1.75 & ; \quad R/t < 4 \\ 0.2625 \cdot R/t + 0.7 & ; \quad 4 \leq R/t \leq 8 \\ 2.80 & ; \quad 8 < R/t \end{cases}$$

The difference of SCF between weld type and bend type is the effect of bending stress due to cross bending occurred in bend type knuckle part. It is noted that the location of hot spot point is different between both structures. If the cross bending is controlled effectively, SCF for bend type becomes almost same as the SCF for weld type. According to the results of FE analyses, $K_3 = 0.9$ was obtained as the average value for weld type, which shows the effect of stress reduction due to the longitudinal rib. And the correction factor K_3 was so determined that the $K_1 \times K_3$ for bend type becomes same as one for weld type. Then the correction factors K_3 for weld type and bend type could be obtained as below:

$$K_{3,weld} = 0.9$$

$$K_{1,bend} = \begin{cases} 0.85 & ; \quad R/t < 4 \\ 1.15 - 0.075 \cdot R/t & ; \quad 4 \leq R/t \leq 8 \\ 0.55 & ; \quad 8 < R/t \end{cases}$$

According to the results of FE analyses, $K_2 = 0.9$ and $K_4 = 0.9$ were obtained as the average values regardless of the difference of knuckle type.

3. Summary

The geometrical stress concentration factor for the bilge hopper knuckle is giving by the following equation.

$$K_t = K_0 \times K_1 \times K_2 \times K_3 \times K_4$$

K_0	;	SCF depending on the dimensions of the considered structure
K_1	;	Correction factor depending on the plate bending process
K_2	;	Correction factor depending on the thickness increment of the web plate
K_3	;	Correction factor depending on the insertion of horizontal gusset or longitudinal rib
K_4	;	Correction factor depending on the insertion of transverse rib

Table 4 Stress concentration factor K_0

Plate thickness (*mm*)	**Angle of hopper slope plate to the horizontal (*deg.*)**			
	40	45	50	90
16	3.0	3.2	3.4	4.2
18	2.9	3.1	3.3	4.0
20	2.8	3.0	3.2	3.8
22	2.7	2.9	3.1	3.6
24	2.6	2.8	3.0	3.5
26	2.6	2.7	2.9	3.4
28	2.5	2.7	2.8	3.3
30	2.4	2.6	2.7	3.2
Note: Values for intermediate plate thickness and angle may be interpolated from the values given in the table.				

Table 5 Correction Coefficients

<table>
<tr><th>Type of knuckle</th><th>K_1</th><th>K_2</th><th>K_3</th><th>K_4</th></tr>
<tr><td>Weld Type</td><td>1.7</td><td rowspan="2">0.9</td><td>0.9</td><td rowspan="2">0.9</td></tr>
<tr><td>Bend Type</td><td>1.75 ; $R/t < 4$
2.80 ; $R/t > 8$</td><td>0.85 ; $R/t < 4$
0.55 ; $R/t > 8$</td></tr>
</table>

Notes :

(1) When evaluating K_1 and K_3 between $4 \le R/t \le 8$, the linear interpolation is applied
"R" denotes the radius of bend part and "*t*" denotes the plate thickness

(2) In using the correction coefficients K_2, the increase in web thickness is taken based on the plate thickness of the inner bottom plating.

(3) In using the correction coefficients K_3 and K_4, the members should be arranged such that the bending deformation of the radius part is effectively suppressed.